A critical state model for overconsolidated structured clays

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1. Introduction

Reconstituted clay behaviour has been studied for the last four decades. The critical state theory is one of the major developments of the constitutive model for clays. The models of the Cam Clay family such as the Cam Clay (CC) model [40] and the Modified Cam Clay (MCC) model [39] are used widely to describe clay behaviour in the reconstituted state. The models give good agreement between experimental and model simulation results. However, recent studies [26,4,27,7,8,43,19] have shown that natural clay is structured and its behaviour is different from its behaviour in the reconstituted state. Cement stabilised clay is also identified as artificially structured clay [33,16,18]. The critical state theory, which is widely accepted for simulating clay behaviour [42,35], has been extended to simulate structured clay behaviour by considering the effect of soil structure [14,21,22,41,30,25]. Taiebat et al. [46] have proposed a model taking into account the effect of destructuring and anisotropy on the stress–strain response.

The Structured Cam Clay (SCC) model [30] is a simple and predictive critical state model for structured clays. It was formulated simply by introducing the influences of soil structure on the volumetric deformation and the plastic deviatoric strain into the MCC model. A key assumption of the SCC model is that both the hardening and destructuring of structured clay are dependent on plastic volumetric deformation. A simple elliptical yield surface with a non-associated flow rule is used for predicting the behaviour of clays in naturally structured states. The SCC model successfully captures many important features (particularly volumetric hardening/destructuring) of the naturally structured clay behaviour with insignificant cohesion.

However, the cohesion and strain softening, which are generally very significant for naturally structured stiff clays [4,21,18], cannot be taken into account by the SCC model. For better simulation of cemented clays, the modified effective concept was introduced into the SCC model [17].

Suebsuk et al. [45] developed a general constitutive model based on the critical state framework for destructured, naturally structured and artificially structured clays. The proposed model is the Modified Structured Cam Clay (MSCC) model. It was formulated based on the SCC model for cemented clay [17]. In the MSCC model, the influence of soil structure and destructuring were introduced into the yield function, plastic potential, and hardening rule to describe the mechanical behaviour of structured clays. The soil structure increases the mean effective yield stress under isotropic compression (p′ y) and structure strength (p′ s). The destructuring mechanism in the MSCC model is the process of reducing the
structure strength, $p_y$ due to the degradation and the crushing of structure. The destructuring behaviour of the MSCC model is divided into two parts for volumetric deformation and shearing. The destructuring due to shearing causes a reduction in structure strength that is directly related to the plastic deviatoric strain, $\varepsilon_d^p$. The strain softening can be described by the MSCC model when the stress states are on the state boundary surface.

In the MSCC model, a yield surface separates elastic behaviour (i.e., stress states inside the yield surface) from elastoplastic behaviour (i.e., stress states on the yield surface). However, in reality, even in the overconsolidated state, naturally structured clays, artificially structured clays and many geosynthetics often exhibit a non-recoverable behaviour upon unloading and repeated loading [4], which results in overestimation of the soil strength and the lack of a smooth transition between elastic and elastoplastic behaviour. To obtain accurate predictions of the deformational response, it is necessary to improve the MSCC model to better simulate the soil behaviour in the overconsolidated state. It is noted herein that the traditional term “overconsolidated state” is also used for structured clay to indicate the relative position of the current stress in relation to its limit state surface or bounding surface that the yield stress (stress history) is caused by both mechanical and chemical effects.

There are two basic theories that have been widely used in modelling the plastic deformation of soils for loading inside the limit surface. One is the multi–surface plasticity (MS) theory, which was originally applied to metal with the kinematic hardening (KH) rule [20,34] and then the Mroz’s multi-surface plasticity concept was applied to soil stress–strain modelling by Prevost (1977, 1978). The other is the bounding surface plasticity (BS) theory, which was formulated with the kinematic hardening rule [13,9,24]. Those concepts have been extended to many soil models for geomaterials (e.g.,[14,42,22,41,38,32,3]). However, the major setback of MS theory and KH rule is that the numerical computation is complex and requires considerable memory for the configuration of the sub-yield and stress reversal surface. The bounding surface model with the radial mapping rule proposed by Dafalias and Herrmann [11] and Dafalias [10] removed the above deficiencies. Many advanced soil models have been developed based on the radial mapping rule [2,47,52,50,51]. This version of bounding surface plasticity is introduced in the current modelling of plastic deformation of structured soil inside the bounding surface.

To keep a model simple for practical use, it is impossible to include all features of the soil behaviour in a model. A complex model with many features should be avoided because it may have hidden inaccuracies, numerical instabilities, lack of a converged solution and other errors [49]. In this paper, the bounding surface theory with radial mapping rule was employed with the MSCC model to simulate the stress–strain behaviour of overconsolidated structured clays because this theory is simple and predictive. The proposed model is called the “Modified Structured Cam Clay with Bounding Surface Theory” (MSCC-B) model. The MSCC-B model is presented in a four-dimensional space consisting of the current stress state, the current voids ratio, the stress history and the current soil structure. The MSCC-B model requires six parameters in addition to the standard parameters from the MCC model. The five parameters ($b, \Delta \varepsilon_v, p_{\theta}^{0}, \xi$ and $\psi$) are the same as those of the MSCC model and have clear physical meaning. A new constant material parameter, $h$, is proposed in the study to take into account the effect of the material characteristics on the plastic hardening modulus in the overconsolidation state.

Finally, the MSCC-B model is evaluated in the light of the model performance. The MSCC-B model was implemented in a single element elastoplastic calculation for simulating the behaviour of structured clays in both naturally and artificially structured states. Test data over a wide range of the mean effective stress and degree of cementation under both drained and undrained shearing were used in the verification.

### 2. MSCC: a generalised critical state model for structured clays

The MSCC model was developed based on the following key assumptions: (1) the reconstituted or destructured soil behaviour can be described adequately by the MCC model with the intrinsic properties obtained from a reconstituted sample, (2) the structured soil behaviour is divided into the elastic and virgin yielding regions by the current yield surface and (3) the destructuring is only related to the plastic strain (insignificant for stress state inside the yield surface).
The material idealisation of structured clay for the MSCC model is shown in Fig. 1. The current yield surface (the structural yield surface) is defined by its current stress state \((q, p')\), stress history \((p_0^0)\) and structure strength \((p_b^0)\). The structural yield surface in the \(q-p'\) plane is assumed to be elliptical in shape and passes through the origin of the stress coordinates. The \(p_b^0\) affects the origin of structural yield surface that moves to the left side of the mean effective stress axis. The structural yield surface is thus given in terms of the conventional triaxial parameters as follows:

\[
f = q^2 - M^2(p' + p_b^0)(p'_b - p') = 0, \tag{1}
\]

where \(p_b^0\) is the reference size of the yield surface and \(M\) is the gradient of failure envelope in the \(q-p'\) plane.

The elastic behaviour inside the yield surface is assumed to be the same as in traditional Cam Clay family. It is defined by the bulk modulus \((K')\) and shear modulus \((G')\). These parameters are determined by the following basic equations:

\[
K' = \frac{(1 + e) p'}{\kappa}, \tag{2}
\]

\[
G' = \frac{3(1 - 2\mu')}{2(1 + \mu')} K', \tag{3}
\]

where \(\kappa\) is a gradient of unloading or swelling line of structured clay, \(\mu'\) is Poisson’s ratio and \(e\) is the voids ratio.

The plastic potential with the influence of soil structure takes the form

\[
ge = q^2 + \frac{M^2}{1 - \psi} \left[ \frac{(p' + p_b^0)}{p'_b + p_b^0} \right]^2 \left( p'_b + p_b^0 \right)^2 - \left( p' + p_b^0 \right)^2 = 0, \tag{4}
\]

where \(p_b^0\) is a reference size parameter and \(\psi\) is a parameter describing the shape of the plastic potential. The \(p_b^0\) can be determined for any stress state \((q, p')\) by solving the above equation. Fig. 2 shows the shape of the plastic potentials, which pass through a stress state \((q_1, p'_1)\).

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Fig. 1. The material idealisation of structured clays for the MSCC model.

Fig. 2. Influence of the \(\psi\) on the shape of plastic potential.

Fig. 3. Idealisation of bounding surface plasticity model for isotropic compression of structured clays.
The voids ratio, which is a state parameter for the derivation of the hardening rule during virgin compression along a general stress path, is proposed as follows:

\[ e = e_{ic} - \kappa \ln \sigma' + \Delta e (\frac{\sigma'}{p_0})^b - (\sigma' - \kappa) \ln p_0. \]  

(5)

where \( b \) is a destructuring index due to the volumetric deformation, \( p_{yc} \) is the initial isotropic yield stress and \( \Delta e \) is an additional voids ratio between the intrinsic compression line (ICL) and the structured compression line (SCL) at the \( p_{yc} \) (Fig. 1a). Eq. (5) shows that as the stress increases, \( \Delta e \) approaches zero. The rate of the removal of soil structure, i.e., represented by \( \Delta e \), is dependent on destructuring index \( b \) (Liu and Carter [28,29]). For the deformation range of experimental data and practical applications concerned, Eq. (5) is valid and effective for determination of volumetric deformation.

Based on the voids ratio in Eq. (5), the plastic volumetric strain increment can be formulated as follows:

\[ \delta e_p = \left[ (\sigma' - \kappa) + \frac{M}{M - \eta} b \Delta e \right] \frac{\delta p_0}{(1 + \epsilon) p_0}. \]  

(6)

The derivation of this equation can be found in the papers by Liu and Carter [30,31]. In the MSCC model, the destructuring law is proposed to describe the reduction in the modified effective stress [45]. Destructuring during shearing consists of two processes: degradation of structure and crushing of soil–cementation structure. The degradation of structure occurs when the stress state is on the yield surface whereas the crushing of the soil–cementation structure happens at post-failure during strain softening. The \( p_b \) is assumed to be constant up to the virgin yielding. During virgin yielding (during which plastic deviatoric strain occurs), the \( p_b \) gradually decreases due to degradation of the structure until the failure state. Beyond this state, a sudden decrease in the \( p_b \) occurs due to the crushing of the soil–cementation structure and diminishes at the critical state. The reduction in \( p_b \) due to the degradation of structure (pre-failure) and the crushing of the soil–cementation structure (post-failure) are proposed in terms of the plastic deviatoric strain as follows:

\[ p_b = p_{b0} \exp(-\varepsilon_d^p) \]  

for pre-failure (hardening and destructuring),

(7)

\[ p_b = p_{b0} \exp \left[ -\zeta (\varepsilon_d^p - \varepsilon_d^{pf}) \right] \]  

for post-failure (crushing),

(8)

where \( p_{b0} \) is the initial structure strength, \( p_{b0} \) is the structure strength at failure (peak strength), \( \varepsilon_d^{pf} \) is the plastic deviatoric strain at failure and \( \zeta \) is the destructuring index due to shearing in...
post-failure. The formulation and parametric study of the material parameters of MSCC have been presented clearly in the paper by Suebsuk et al. [45].

3. MSCC-B: a critical state model for overconsolidated structured clays

In 1975, Dafalias and Popov firstly proposed the concept of bounding surface plasticity. Then a bounding surface theory with radial mapping rule was proposed by Dafalias and Herrmann [11] and Dafalias [10]. This version of bounding surface plasticity, i.e., the bounding surface model with radial mapping rule, is introduced in the modelling of plastic deformation of structured soil inside the bounding surface. For structured clay, the bounding surface is referred to as the structural bounding surface, which is affected by the soil structure. The variation of the structural bounding surface is dependent on destructuring as well as hardening, both of which are assumed to be determined due to the change in plastic volumetric deformation and plastic deviatoric deformation following that of classic plasticity [42,31].

Based on the bounding surface theory with the radial mapping rule and the framework of the MSCC model, Fig. 3 shows the schematic diagram for illustrating the change of stress state (\( e - \ln p' \) plane and \( q - p' \) plane) under isotropic compression loading for path 1–6. Point 1 is an initial in situ state for a clay whose structural bounding surface is defined by isotropic yield stress on SCL, \( p_{y0} \) and structure strength, \( p_s \). As loading continues along stress path 1–2, subyielding occurs, and the loading surface expands (inside the structural bounding surface). Supposing that the loading surface and the structural bounding surface coincide at point 2, that is, \( p_{c2} = p_{y0} \) (where \( p_{c2} \) and \( p_{y0} \) are the reference size of the loading and structural bounding surfaces at point 2, respectively), virgin yielding commences at this point and continues along the path 2–3.

Unloading starts at point 3 and continues along the stress path 3–4, i.e., \( \phi p'_{c} < 0 \) and \( p'_{c} \neq p'_{y0} \). Subsequently, the current loading surface enters inside the structural bounding surface. When the stress path changes direction again at point 4 with \( \phi p'_{c} > 0 \).

### Table 1
Parameters for a parametric study on the effect of \( h \).

<table>
<thead>
<tr>
<th>Parameter/symbol</th>
<th>Physical meaning</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>Gradient of ICL in ( e - \ln p' ) plane</td>
<td>0.147</td>
<td>–</td>
</tr>
<tr>
<td>( e_c' )</td>
<td>Voids ratio at reference stress (( p' = 1 ) kPa) on ICL</td>
<td>1.92</td>
<td>–</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Gradient of unloading-reloading line of SCL in ( e - \ln p' )</td>
<td>0.027</td>
<td>–</td>
</tr>
<tr>
<td>( b )</td>
<td>Non-dimension parameter describing the destructuring by volumetric plastic strain</td>
<td>0.6</td>
<td>–</td>
</tr>
<tr>
<td>( \Delta e_r )</td>
<td>Additional of voids ratio between ICL and SCL at yield stress</td>
<td>1.92</td>
<td>–</td>
</tr>
<tr>
<td>( p_{y0} )</td>
<td>Mean effective yield stress on SCL</td>
<td>100 kPa</td>
<td></td>
</tr>
<tr>
<td>( G' )</td>
<td>Gradient of critical state line in ( q - p' ) plane</td>
<td>1.2</td>
<td>–</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Non-dimension constant parameter define the shape of plastic potential</td>
<td>2.0</td>
<td>–</td>
</tr>
<tr>
<td>( \beta^{(0)} )</td>
<td>Initial structure strength on in ( q - p' ) plane</td>
<td>20 kPa</td>
<td></td>
</tr>
<tr>
<td>( \zeta )</td>
<td>Non-dimension constant parameter describing the destructuring by shearing</td>
<td>10</td>
<td>–</td>
</tr>
</tbody>
</table>

### Table 2
Physical properties of the base clays.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Pappadai clay</th>
<th>Ariake clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific density</td>
<td>2.73</td>
<td>2.70</td>
</tr>
<tr>
<td>Natural water content (%)</td>
<td>29.8–32.6</td>
<td>135–150</td>
</tr>
<tr>
<td>Liquid limit (%)</td>
<td>65</td>
<td>120</td>
</tr>
<tr>
<td>Plasticity index (%)</td>
<td>35</td>
<td>63</td>
</tr>
<tr>
<td>Activity</td>
<td>0.42–0.72</td>
<td>1.24–1.47</td>
</tr>
<tr>
<td>Clay fraction (%)</td>
<td>50</td>
<td>55</td>
</tr>
<tr>
<td>Silt fraction (%)</td>
<td>–</td>
<td>44</td>
</tr>
<tr>
<td>Sand fraction (%)</td>
<td>–</td>
<td>1</td>
</tr>
<tr>
<td>In-situ voids ratio</td>
<td>0.68–0.90</td>
<td>3.65–4.05</td>
</tr>
<tr>
<td>Sample type</td>
<td>Intact</td>
<td>Cemented</td>
</tr>
</tbody>
</table>

### Table 3
Testing programme for triaxial tests on the base clays.

<table>
<thead>
<tr>
<th>Base clay</th>
<th>Sample No.</th>
<th>YSR iso</th>
<th>( p' ) before shearing (kPa)</th>
<th>( e ) before shearing</th>
<th>Shearing condition</th>
<th>Strain or displacement rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intact Pappadai clay</td>
<td>TN14</td>
<td>4.52</td>
<td>500</td>
<td>0.875</td>
<td>D</td>
<td>0.2–0.6%/day</td>
</tr>
<tr>
<td></td>
<td>TN15</td>
<td>2.825</td>
<td>800</td>
<td>0.8725</td>
<td>D</td>
<td>0.18%/day</td>
</tr>
<tr>
<td></td>
<td>TN18</td>
<td>1.50</td>
<td>1500</td>
<td>0.839</td>
<td>D</td>
<td>0.4%/day</td>
</tr>
<tr>
<td></td>
<td>TN17</td>
<td>1.00</td>
<td>2500</td>
<td>0.766</td>
<td>D</td>
<td>0.189%/day</td>
</tr>
<tr>
<td></td>
<td>TN5</td>
<td>3.22</td>
<td>700</td>
<td>0.895</td>
<td>U</td>
<td>4%/day</td>
</tr>
<tr>
<td></td>
<td>TN3</td>
<td>2.167</td>
<td>1042</td>
<td>0.862</td>
<td>U</td>
<td>4.3%/day</td>
</tr>
<tr>
<td></td>
<td>TN11</td>
<td>1.40</td>
<td>1600</td>
<td>0.856</td>
<td>U</td>
<td>5%/day</td>
</tr>
<tr>
<td>Cemented Ariake clay</td>
<td>AW6-50</td>
<td>1.40</td>
<td>50</td>
<td>4.098</td>
<td>D</td>
<td>0.0025’’</td>
</tr>
<tr>
<td></td>
<td>AW18-400</td>
<td>5.00</td>
<td>400</td>
<td>3.673</td>
<td>D</td>
<td>0.0025’’</td>
</tr>
<tr>
<td></td>
<td>AW18-500</td>
<td>4.00</td>
<td>500</td>
<td>3.658</td>
<td>D</td>
<td>0.0025’’</td>
</tr>
<tr>
<td></td>
<td>AW18-1000</td>
<td>2.00</td>
<td>1000</td>
<td>3.657</td>
<td>D</td>
<td>0.0025’’</td>
</tr>
<tr>
<td></td>
<td>AW9-100</td>
<td>3.70</td>
<td>100</td>
<td>3.650</td>
<td>U</td>
<td>0.0075’’</td>
</tr>
<tr>
<td></td>
<td>AW9-200</td>
<td>1.85</td>
<td>200</td>
<td>3.589</td>
<td>U</td>
<td>0.0075’’</td>
</tr>
<tr>
<td></td>
<td>AW18-200</td>
<td>10.00</td>
<td>200</td>
<td>3.707</td>
<td>U</td>
<td>0.0075’’</td>
</tr>
<tr>
<td></td>
<td>AW18-400</td>
<td>5.00</td>
<td>400</td>
<td>3.673</td>
<td>U</td>
<td>0.0075’’</td>
</tr>
</tbody>
</table>

Remarks: U = undrained test and D = drained test.

\( \frac{\Delta e}{\Delta t} \), mm/min.
reloading then commences and continues along stress path 4–5. The perfectly hysteresis loop is assumed for the MSCC-B model, and hence $p_{c3} = p_{c5}$. At point 5, the loading surface coincides with the structural bounding surface, i.e., $p_{c5} = p_{03}$. In this case, virgin yielding recommences at point 5 and continues for loading along stress path 5–6. For isotropic condition, the expansion and contraction of the loading surface is only dependent on the plastic deformation. The change of stress state under shearing can be explained in the same way, but $p_{0c}$ decreases due to the destructuring law (Eqs. (7) and (8)). The plastic deviatoric strain influences the stress path direction when $\eta > 0$.

### Table 4

Parameters for MSCC-B model for intact Pappadai clay.

<table>
<thead>
<tr>
<th>Test type</th>
<th>Parameter/symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isotropic compression test</td>
<td>$\lambda$</td>
<td>–</td>
<td>0.206</td>
</tr>
<tr>
<td></td>
<td>$e_{IC}$</td>
<td>–</td>
<td>3.17</td>
</tr>
<tr>
<td></td>
<td>$k$</td>
<td>–</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>–</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>$\Delta e_i$</td>
<td>–</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>$p_c$</td>
<td>kPa</td>
<td>2300</td>
</tr>
<tr>
<td>Drained or undrained shearing test</td>
<td>$h$</td>
<td>–</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>$M$</td>
<td>kPa</td>
<td>40,000</td>
</tr>
<tr>
<td></td>
<td>$p_{c0}$</td>
<td>kPa</td>
<td>480</td>
</tr>
<tr>
<td></td>
<td>$\xi$</td>
<td>–</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>$\psi$</td>
<td>–</td>
<td>2.0</td>
</tr>
</tbody>
</table>

* Tested result on reconstituted sample.

### Table 5

Parameters for MSCC-B model for cemented Ariake clay.

<table>
<thead>
<tr>
<th>Test type</th>
<th>Parameter/symbol</th>
<th>Unit</th>
<th>$A_w = 6%$</th>
<th>$A_w = 9%$</th>
<th>$A_w = 18%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isotropic compression test</td>
<td>$\lambda$</td>
<td>–</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>$e_{IC}$</td>
<td>–</td>
<td>4.37</td>
<td>4.37</td>
<td>4.37</td>
</tr>
<tr>
<td></td>
<td>$k$</td>
<td>–</td>
<td>0.06</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>–</td>
<td>0.05</td>
<td>0.01</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>$\Delta e_i$</td>
<td>–</td>
<td>2.0</td>
<td>2.5</td>
<td>2.72</td>
</tr>
<tr>
<td></td>
<td>$p_c$</td>
<td>kPa</td>
<td>70</td>
<td>370</td>
<td>2000</td>
</tr>
<tr>
<td>Drained or undrained shearing test</td>
<td>$h$</td>
<td>–</td>
<td>100</td>
<td>800</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>$M$</td>
<td>–</td>
<td>1.44</td>
<td>1.73</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>$G$</td>
<td>kPa</td>
<td>4000</td>
<td>8000</td>
<td>40,000</td>
</tr>
<tr>
<td></td>
<td>$p_{c0}$</td>
<td>kPa</td>
<td>50</td>
<td>75</td>
<td>730</td>
</tr>
<tr>
<td></td>
<td>$\xi$</td>
<td>–</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>$\psi$</td>
<td>–</td>
<td>0.9</td>
<td>0.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

* Tested result on reconstituted sample.

3.1. Bounding surface theory with radial mapping rule

The bounding surface theory for modelling the elastoplastic deformation of material with the radial mapping rule [47,52] was extended for this study. The structural bounding surface is defined by the stress history, $p_c$, and structure strength, $p_0$. The clay behaviour is divided into the virgin yielding ($p_c' = p_{0c}$) and subyielding ($p_c' < p_{0c}$), where $p_c'$ is a reference size of loading surface. Similar to the MSCC model, the equation for the structural bounding surface is assumed to be elliptical and can be expressed as follows:

$$ f = q^2 - M^2(p + p_b)(p_{0c} - p) = 0. $$ (9)

The loading surface is defined as the surface on which the current stress state remains. For simplicity, the loading surface is assumed to be the same shape as the structural bounding surface and with an aspect ratio similarly equal to $M$.

3.1.1. Plastic potential

The plastic potential adopted in the MSCC-B model is the same as that proposed in the MSCC model as shown in Eq. (4).

3.1.2. Destructuring law

The volumetric hardening and destructuring law presented in the previous section (Eq. (6)) were adopted in the formulation of the hardening modulus for the MSCC-B model with the assumption that $p_{0c} = p_c$ and $\delta p_{0c} = \delta p_c$. The destructuring law due to shearing presented in a previous section (Eqs. (7) and (8)) is used in the MSCC-B model.

3.1.3. Mapping rule

The mapping rule is an essential part of a bounding surface theory based on which the stiffness of soil deformation during subyielding state is defined. A detailed discussion on the significance of mapping rule can be found in a paper by Yang et al. [51]. The original radial mapping rule was modified for structured clay in this study since this method is both simple and effective for many conventional stress paths. The image stress point on the structural bounding surface is used for determining the plastic volumetric strain. The schematic diagram for the mapping rule is shown in Fig. 4. An association between any stress point and the image stress point is described by the intersection of the structural bounding surface with the straight line passing through the origin and the current stress state, based on the assumption that the hardening modulus at the current stress point ($H$) is related to the hardening modulus at its corresponding image stress point ($H_I$) as well as to the ratio of the image stress ratio ($\chi$). The variation of $\chi$ is assumed to be a function of both the current stress state ($q, p'$) and the image stress state ($q_I, p'_I$) as follows:

![Fig. 7. Comparison of measured and simulated isotropic compression tests of intact Pappadai clay.](image7)

![Fig. 8. Comparison of measured and simulated isotropic compression tests of cemented Ariake clay.](image8)
The parameter $\alpha$ becomes the unique value ($\alpha = \frac{p}{p_0} = \frac{q}{q_0} = \frac{p}{p_0}$) at the critical state when $p_0$ is zero due to the complete removal of cementation structure (destructuring).

### 3.2. Hardening modulus

#### 3.2.1. Hardening modulus at an image stress point ($H_j$)

Based on the state boundary surface theory, the variation of the structural bounding surface ($dp_{eq}$) is related to the plastic volumetric strain increment ($\delta \sigma_v^p$). It is developed based on Eq. (6) as follows:

$$\delta p_{eq} = \frac{(1+e)p_0}{(\lambda - \kappa) + \left(\frac{M}{M - \eta}\right) b \Delta e} \delta \sigma_v^p$$  \hspace{1cm} (11)

The differential form of the structural bounding surface is

$$\frac{df}{dp'} \delta p' + \frac{df}{dq} \delta q + \frac{df}{dp_0} \delta p_0 = 0.$$  \hspace{1cm} (12)

Differentiating Eq. (9) with respect to $p_0$,

$$\frac{df}{dp_0} = -M^2 (p' + p_0).$$  \hspace{1cm} (13)

Substituting Eqs. (11) and (13) into Eq. (12),

$$\frac{df}{dp'} \delta p' + \frac{df}{dq} \delta q + \frac{df}{dp_0} \delta p_0 = \frac{(1+e)p_0}{(\lambda - \kappa) + \left(\frac{M}{M - \eta}\right) b \Delta e} \delta \sigma_v^p = 0.$$  \hspace{1cm} (14)

The plastic strain increment in the subyielding state is defined in terms of the hardening modulus, yield function and plastic potential in the same way as that in the virgin yielding state as previously successfully done by Dafalias and Herrmann [11], Bardet [2] and Yu et al. [52]. Thus,

$$\delta \sigma_v^p = \frac{1}{H_j} \left(\frac{df}{dp'} \delta p' + \frac{df}{dq} \delta q \right) \left| \frac{\partial g}{\partial p} \right|.$$  \hspace{1cm} (15)

Substituting Eq. (15) into Eq. (14),

$$H_j = M^2 (p' + p_0) \left(\frac{1+e}{p_0} \right) \left(\frac{M}{M - \eta}\right) b \Delta e \left| \frac{\partial g}{\partial p} \right|.$$  \hspace{1cm} (16)

where $\frac{\partial g}{\partial p}$ are given as

$$\frac{\partial g}{\partial p} = \frac{M^2}{1 - \psi} \left( \frac{2 (p_1 + p_2) (p_0 + p_0)^2}{\psi (p_1 + p_0)} - 2 (p_0 + p_0) \right).$$  \hspace{1cm} (17)

#### 3.2.2. Hardening modulus at the stress point ($H$)

A specific feature of the bounding surface theory is that the hardening modulus ($H$) is not only dependent on the location of
the image point but also on a function of the distance from the stress point to the structural bounding surface with the following requirements [12,23,52]:

\[
H = \begin{cases} 
+\infty & \text{if } \alpha = 0, \\
H_j & \text{if } \alpha = 1.
\end{cases}
\] (18a) (18b)

The restriction imposed by Eqs. (18a) and (18b) ensures that the clay behaviour is almost purely elastic when the stress state is far away from the structural bounding surface and that the stress point and the structural bounding surface move together when the current stress state lies on the structural bounding surface. The hardening modulus for the MSCC-B model under monotonic loading condition is proposed to satisfy the restriction Eqs. (18a) and (18b) as follows:

\[
H = H_j + (h \cdot H_j) \cdot \frac{(1 - \alpha)^2}{\alpha},
\] (19)

where \( h \) is the non-dimensional parameter describing the effect of material characteristics on \( H \) and \( H_j \) is the initial hardening modulus at the image stress point, which is calculated by Eq. (16).

3.3. Model parameters

There are 12 parameters for the MSCC-B model. Six parameters (\( k, e, \kappa, M, p_0, G \) or \( \mu' \)) are the basic parameters adopted from the MCC model. Five parameters (\( \beta_0, p_{00}, c, \xi \) and \( \psi \)) are the structural parameters describing the hardening and destructuring behaviour for the original MSCC model. The last parameter is \( h \), which is newly presented in this paper to describe the effect of the material characteristics on the hardening modulus in the overconsolidated state. The parameters denoted by a superscript * are the parameters tested with reconstituted samples [4]. The details of the parameter determination can be found in the papers by Horpibulsuk et al. [17] and Suebsuk et al. [45].

The effect of \( h \) on the shear behaviour is illustrated in Figs. 5 and 6 using the model parameters listed in Table 1. Fig. 5a shows the effect of \( h \) on the undrained stress path. It is clear that \( h \) significantly affects the hardening and destructuring behaviours for structured clay in the overconsolidated state. An increase in \( h \) increases both the stiffness and strength of the clay, as shown in Fig. 5a and b. The smoothness of the excess pore pressure–deviatoric strain curve is illustrated in Fig. 5c for various \( h \). At the same deviatoric stress, a decrease in \( h \) leads to a larger deformation produced by the shearing. Fig. 6 presents the influence of \( h \) on the relationship between \( \alpha - \varepsilon_d \) under drained shearing. The parameter \( h \) insignificantly affects the soil stiffness when it is smaller than 1.

The parametric studies (Figs. 5 and 6) show that the destructuring depends on both \( \varepsilon_p \) and \( \varepsilon_d \). Its behaviour can be predicted by Eqs. (6)–(8). The plastic volumetric strain occurs at the early stage of loading. If \( \alpha < 1 \), the stress state stays inside the structural bounding surface, and the destructuring is caused by the change in plastic volumetric strain due to \( \Delta p \) and the degradation of soil structure due to the deviatoric strain. The destructuring gradually occurs as shown by the smooth stress–strain relationship. When \( \alpha = 1 \), the loading surface and structural bounding surface coincide, and the MSCC-B and MSCC models are the same.

4. Performance of the MSCC-B model

In this section, the MSCC-B model is verified by a comparison between measured and simulated data. The tested results of intact
Pappadai clay [6,8] and cemented Ariake clay [15,18] were used for this verification.

The physical properties of both clays are presented in Table 2. The testing programmes of the Pappadai and cemented Ariake clays are summarised in Tables 3. The model parameters were obtained with the following steps:

(a) The isotropic compression test results of structured samples and reconstituted (remoulded) samples were taken to determine compression parameters. The $\kappa$ and $e^\prime/C_3$ were determined from the intrinsic compression line, ICL [4]. The three additional structural parameters ($p'_0$, $\Delta e$, and $b$) were determined from the structured compression line, SCL [30,45]. The $\kappa$ was determined from the unload–reloading line of the structured sample.

(b) Undrained and drained shearing test results of isotropically consolidated structured clay were taken to determine the parameters ($M$, $p'_0$, $G$, $\psi$, $\xi$, and $h$) at various isotropic yield stress ratios (YSRiso). The gradient of critical state line $M$ and initial structure strength $p'_0$ was obtained from the stress path (vide Fig. 1b). The shear modulus $G$ was approximated from the $q$–$e'$ curve. The parameters $\psi$ and $\xi$ are dependent on the type of clays and the degree of cementation [45]. The parameter $\psi$ were estimated from the shape of the structural bounding surface. For natural clay, the plastic potential is approximately elliptical, and $\psi$ can be taken as 2. For cemented clay, it is usually less than 2 [45]. The parameter $\xi$ was estimated from the stress–strain response, and its value is $1 < \xi < 100$ [45]. The parameter $h$ was obtained by plotting the simulated $a$–$e'$ curve compared with measured data.

Tables 4 and 5 present the input parameters selected for intact Pappadai clay and cemented Ariake clay, respectively.

4.1. Isotropic compression

Fig. 7 shows the comparison between the measured and simulated data for the isotropic compression of intact Pappadai clay. The tested data can be captured well by the model simulation. The smooth $e^\prime$–$\ln p'$ curve for pre- and post-yield states is seen. The meta-stable voids ratio, which cannot be degraded with the increase in confining stress, is predicted by the MSCC-B model. The lower the $b$, the smaller the destructuring. Fig. 8 compares the model simulation results of the isotropic compression curves with the measured data for the cemented Ariake clay at different cement contents. The simulations were performed with a single set of intrinsic parameters. The results show that samples with higher cement content exhibit smaller destructuring, which is controlled by $b$. The MSCC-B model simulation broadly matches the experimental results better than the original MSCC model, which cannot simulate the subyielding behaviour. From the comparisons, it is seen that the destructuring by shearing does not affect the isotropic compression response, and thus the simulation of isotropic compression is only controlled by six parameters ($\kappa$, $e^\prime/C_3$, $\Delta e$, $p'_0$, $\psi$, and $h$).

4.2. Drained triaxial shearing

The results of the model simulation of isotropically consolidated drained compression (CID) shearing of the intact Pappadai clay are shown in Fig. 9. For moderate to large deviatoric strain
(\eta_d = 0.1–20\%), a constant shear modulus is assumed throughout
this simulation. The MSCC-B model gives more realistic predictions
than those predicted by the original MSCC model. The smooth
stress–strain relationship can be captured by the MSCC-B model.
By comparison of the stress ratio–deviatoric strain curve and volu-
metric strain–deviatoric strain curve, it is seen that the MSCC-B
model provides a good simulation.

Figs. 10 and 11 present the comparison of measured and simu-
lated data for the cemented Ariake clay at different cement con-
tents. The test data covers the range of cement contents from 6%
to 18\% with the variation of confining stress from 50 kPa to
1000 kPa. The smooth stress–strain response was simulated along
the loading path. The MSCC-B model gives good results for the
stress–strain curve and the volumetric deformation curve.

4.3. Undrained triaxial shearing

Fig. 12 compares the measured and simulated results of the in-
tact Pappadai clay with various YSRiso from 7.50 to 1.50. Figs. 13
and 14 compare the measured and simulated results of the cemen-
ted Ariake clay at various YSRiso and cement contents. It is found
that the deviatoric stress and excess pore pressure versus deviator-
ic strain can be predicted well by the MSCC-B model.

From the simulation of compressibility and shearing for natu-
really and artificially structured clays, it is concluded that the
MSCC-B model can describe the influence of artificial cementation
structure on soil behaviour.

4.4. Performance of the MSCC-B model compared with the original
MSCC model

To demonstrate the improvement of MSCC-B model, simula-
tions of structured soil behaviour made by the MSCC-B model
are compared with those of the original MSCC model as shown in
Figs. 15–17 for Pappadai clay. It is seen that there are the following
improvement in model simulation for the MSCC-B model.

1) The plastic deformation of soil within the yield surface is
captured. This is especially important for highly overconsol-
solidated soils.

2) The peak strength and the smooth transition from hardening
to softening behaviours, for both drained or undrained situ-
ations, have been better represented by the boundary sur-
face theory. This improves the overall accuracy of stress
and strain relationship.
5. Conclusion

Based on an extensive review of the available experimental data, a critical state model with bounding surface theory to describe the mechanical behaviour of naturally and artificially structured clays in the overconsolidated state was proposed. A fundamental hypothesis of the development is that the hardening and destructuring of structured clay are dependent on both the plastic volumetric strain and deviatoric strain. The key feature of the MSCC-B model is that the plastic deformation for stress state inside the yield surface can be well-captured.

The hardening modulus for structured clay was formulated based on the bounding surface theory with the radial mapping rule. It is suitable for describing the compression and shearing behaviours of natural clays and cement-stabilised clays under monotonic loading in the overconsolidated state. The MSCC-B model can simulate the stress–strain-strength relationships of structured clays well over a wide range of YSRiso. Generally speaking, a reasonable good agreement between the model performance and experimental data is achieved. The model can be used as a powerful tool for simulating structured clay behaviour in the over-consolidated state.

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